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Functional ANALYSIS

▦ The set of all bounded continuous real functions defined on the closed unit interval has a metric, defined as

$$\|f\| = \sup\{|f(x)| : x \in [0,1]\}$$

which is also written as

$$\|f\| = \sup|f(x)|$$

and $d(f,g) = \|f-g\| = \sup|f(x) - g(x)|$

This metric space is denoted as $\mathcal{C}[0,1]$.

▦ Linear space :- Let L be a non-empty set. Let each pair of elements x, y of L can be combined by a process called addiban to yield an element z in L, denoted as $z = x+y$.

Also, let $x+y = y+x$ and $x+(y+z) = (x+y)+z$.

Let, there exists in L, a unique element denoted by 0, called zero element or origin such that $x+0 = x \ \forall\, x \in L$

Again, let to each element $x \in L$, $\exists$ a unique element $-x$ (negative of x) such that $x+(-x) = 0$

Also, let the system of real numbers or complex numbers as scalars.

Let, a scalar α and an element $x \in L$ can be combined by a process called SCALAR MULTIPLICATION to yield an element y in L denoted as

$y = \alpha x$ such that

(i) $\alpha(x+y) = \alpha x + \alpha y$

(ii) $(\alpha+\beta)x = \alpha x + \beta x$

(iii) $(\alpha\beta)x = \alpha(\beta x)$

(iv) $1 \cdot x = x$.

The algebraic system L defined by these operations and axioms is called a linear space.

Real linear space :- when scalars are real numbers only.

Complex linear space :- when scalars are complex numbers only.

Normal linear space :- It is a linear space on which there is defined a norm, ⇒ a function which assigns to each element x in the space a real number $\|x\|$ in such a manner that

(i) $\|x\| \geq 0$

(ii) $\|x\| = 0 \Leftrightarrow x = 0$

(iii) $\|x+y\| \leq \|x\| + \|y\|$

(iv) $\|\alpha x\| = |\alpha| \, \|x\|$.

Here, element x is a vector and α is a scalar.